**Grassman Numbers**

Grassman numbers are ones which anti-commute, like fermion operators. A consequence of this is that any Grassman number squared is equal to 0.



As a consequence, a Grassman squared is zero, since,



But Grassmans do commute with complex numbers. In this sense they can be considered a non-zero root of zero. Complex Grassmans can be constructed just like complex numbers:



Note that the modulus of a Grassman is not zero, since:



And also observe,



So a Grassman anticommutes with its complex conjugate. We can verify that a complex Grassman squared is still 0.



As a consequence of the fact that ψ2 = 0, analytic functions of Grassmans are at most linear. Since,



So for example,



Now we’re in position to take derivatives. We’ll presume the obvious, that differentiation is linear and moreover that:



What about integration? Apparently we have to relinquish any Riemann sum association with integration. It doesn’t seem to be the inverse of differentiation either. If we presume integration is linear then all we need to know is the two results:



One condition we impose is that the result be invariant to shifts: ψ → ψ + η. Not sure why – apparently has to do with fact that we want it to accommodate a path integral representation of GF’s that matches our expectations vis a vis the diagrammatic expansion. So this implies:



I don’t see any sources readily justifying the result for the ∫dψ ψ integral. So… we’ll just state the result:



So we see that integration and differentiation are identical, in fact. Now say we have a function of two Grassman numbers: f(ψ1, ψ2). We can expand a function of two variables in similar fashion. The most general result would be:



We can take derivatives and integrals, but general rule is that the differential/integral operator must be touching the variable to generate the result above. May have to permute the #’s to get them into the requisite form. For instance,



where the – sign comes from the need to permute the two ψ’s to get the ∂/∂ψ2 operator touching the ψ2. An integral example:



Now let’s take a look at a Gaussian integral,



If we consider more general functions like,



And so something like:



which is a familiar result.